

## Probability midterm exam (Model 1)

### Question (1):

(A) Box I containing 3 red and 2 blue marbles while Box II contains 2 red and 8 blue marbles. A fair coin is tossed. If the coin turns up heads a marble is chosen from Box I; if it turns up tails a marble is chosen from Box II find the probability that a red marble is chosen?

(B) How many 4-digit numbers can be formed with the 10 digits 0, 1, 2, 3, ..., 9 if

(a) Repetitions are allowed (b) Repetitions are not allowed (c) The last digit must be zero and Repetitions are not allowed?

### Question (2):

The joint density function of two continuous random variables X and Y is

$$f(x, y) = \begin{cases} cxy & 0 < x < 4, 1 < y < 5 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of constant C.

(b) Find  $P(X \geq 3, Y \leq 2)$ .

(c) Find  $P(1 < X < 2, 2 < Y < 3)$ .

(d) Find the marginal distribution function of X.

(e) Find the marginal distribution function of Y.

### Question (3):

(A) If  $X^* = (X - \mu) / \sigma$  is a standardized random variable, Prove that (a)  $E(X^*) = 0$ , (b)  $\text{Var}(X^*) = 1$ ?

(B) Suppose that the two random variables X and Y have joint density function for:

$$f(x, y) = \begin{cases} xy/96 & 0 < x < 4, 1 < y < 5 \\ 0 & \text{otherwise} \end{cases}$$

Find (a)  $E(X)$ , (b)  $E(Y)$ , (c)  $E(XY)$ , (d)  $E(2X + 3Y)$ .